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Numerical study of the gas–liquid–solid flow in hydrocyclones with different configuration of vortex finder

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Abstract

This paper presents a numerical study of the gas–liquid–solid flow in hydrocyclones with different shaped vortex finder. It is done based on the mathematical model we developed recently. In the model, the turbulent flow of gas and liquid is modelled using the Reynolds Stress Model, and the interface between the liquid and air core is modelled using the Volume of Fluid multiphase model. The results are then used in the simulation of particle flow described by the stochastic Lagrangian model. The flow features are examined in terms of flow field, pressure drop, split ratio reported to the underflow, particle trajectories and separation efficiency. The model is validated by the good agreement between the measured and predicted results, and here used to study the effect of vortex finder geometry. The results show that separation efficiency decreases for fine particles but increases for relatively coarse particle as vortex finder length decreases. A thin vortex finder is helpful to high separation efficiency, particularly for coarse particles, but results in an increased pressure drop. In particular, the computational results demonstrate that a short circuit flow exists along the outer wall of vortex finder, resulting in a decrease in separation efficiency. To overcome this problem, a new design is proposed for vortex finder and shown to be able to improve the performance of hydrocyclone considerably.

Keywords: Hydrocyclone; Computational fluid dynamics; Vortex finder; Multiphase flow

1. Introduction

Hydrocyclones are widely used in many industries, particularly in mineral and chemical processing because of its simplicity in design, high capacity, low maintenance and operational cost, and the small physical size of the device. It is generally known that the operational and geometrical parameters determine the performance of a hydrocyclone. The operational conditions, such as feed rate, feed solid concentration and so on, depend on the needs of individual applications. Selection of geometrical parameters is often the first step in designing or utilising hydrocyclones. Therefore, understanding their effects on the performance of a hydrocyclone is important. In particular, the inner geometrical variables describing the vortex finder play a critical role in defining the flow field in a hydrocyclone, including the pattern of the outer and inner spiral flows. They significantly affect the performance of hydrocyclones in terms of flow capacity, separation efficiency, pressure drop, split ratio and so on.

In the past, many empirical equations and models have been formulated to describe hydrocyclone performance [1–3]. However, the fundamentals governing the complicated three-dimensional multiphase flow in hydrocyclones are poorly understood. Therefore, in recent years many efforts have been made to study experimentally the flow within a hydrocyclone and the effect of vortex finder dimension. Knowles et al. [4] used high-speed movies of anisole droplets moving through a hydrocyclone to determine the velocity of liquid flow. More recently, a number of investigators reported their measurements using Laser Doppler Velocimetry (LDV) [5–7]. Kim and Lee [8] showed that the diameter of a vortex finder affected the collection efficiency and pressure drop of cyclones, and proposed an energy-effective cyclone design. Moore and McFarland [9] also tested the performance of cyclones with six different vortex finders. Zhu and Lee [10] depicted the effects of the vortex finder length on the particle collection efficiency using seven different cyclones. Lim et al. [11] evaluated the collection efficiencies of cyclones with 10 different vortex finders. However, such an experimental method

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is often difficult and expensive, largely only suitable for laboratory scale studies rather than for industrial tests and cyclone design.

In the past two decades, significant progress has been made in the mathematical modeling of hydrocyclone process based on computational fluid dynamics (CFD) [12–15]. There are unfortunately various deficiencies associated with the previous CFD models. For example, the nature of air core is often not considered, with assumptions made to simplify the formation and behaviour of air core in a hydrocyclone. This would limit the application of the CFD model to industrial design of hydrocyclones. Recently Wang et al. [16] reported a more comprehensive CFD model. These authors showed that the performance of a hydrocyclone including the separation efficiency, pressure drop and split ratio can be predicted accurately. Their model offers a convenient way to examine the effects of variables related to geometrical conditions on the flow field and particle collection characteristics in hydrocyclones. In fact, it has been used to study the effects of geometrical parameters related to external body dimensions [17].

The purpose of this paper is to extend that work by examining the effects of internal body variables such as vortex finder diameter and length on hydrocyclone performance. The working principle of vortex finder will be explored by analyzing the internal flow in a hydrocyclone. On this basis, a new design is proposed for the vortex finder for hydrocyclones.

2. Mathematical model

2.1. Model description

The multiphase flow in a hydrocyclone is quite complicated, and different treatments may have to be used for different phases. In this work, The fluid flow is modelled as turbulent, described by the Reynolds Stress Model (RSM), the interface between the liquid and air core is modeled by the Volume of Fluid (VOF) model, and the results are then used in the simulation of particle flow described by the stochastic Lagrangian model. Details and justification of the use of these models can be found elsewhere [16,17].

2.2. Simulation conditions

Fig. 1(a) shows the most common hydrocyclone design, which consists of eight main geometrical parameters: diameter of the cylindrical body (D_c), diameter of inlet (D_i), diameter of vortex finder (D_o), diameter of apex (D_u), length of cylindrical part (L_c), length of vortex finder (L_v), thickness of vortex finder (D_v) and included angle (a). These parameters characterize the separation efficiency, pressure drop and split ratio of a hydrocyclone [1,2]. The effects of outer geometrical parameters, which are composed of the body size including the diameter and length, have been studied in the previous work [17]. The effects on the performance were quantified via the CFD model. The inner geometry, i.e. the size and length of vortex finder, plays a critical role in defining the flow field and significantly affects the hydrocyclone performance. The aim of this work is to quantify its effect on the gas–liquid–solid flow and performance of hydrocyclones.

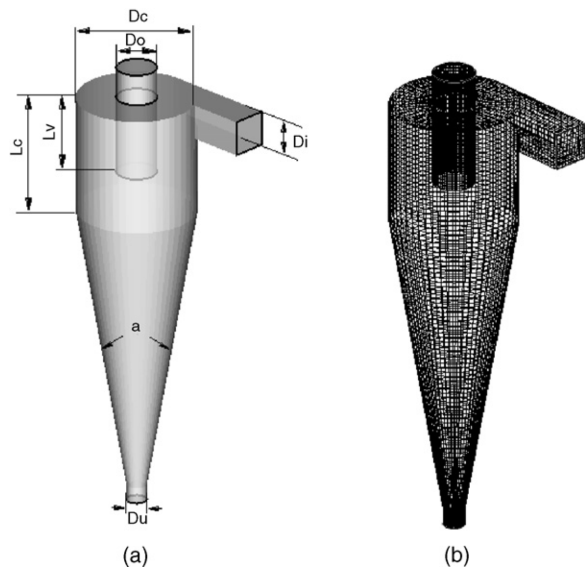

Fig. 1. Schematic (a) and grid (b) representation of the original hydrocyclone considered.

In order to compare the predicted results with those reported elsewhere [7,16], the base hydrocyclone is the same. Table 1 lists the geometry. The pressure at the two outlets (vortex finder and apex) is 1 atm, and the inlet water velocity and particle velocity are both 2.25 m/s. Limestone particles with a density of 2700 kg/m^3 are injected at the inlet. The volume percentage of solids in the feed is less than 10%. Fig. 1(b) shows the computational domain of the original model. The whole computational domain is divided by unstructured hexahedron grids. In the vicinity of the walls and vortex finder, the grid is refined. Preliminary tests confirm that numerical solution is in the range not depending on the characteristics of the mesh size. A “velocity inlet” boundary condition is used at the cyclone inlet, and both outlets use the “pressure-outlet” condition.

The effect of changing the vortex finder dimensions is examined through a parametric study. The length of the vortex finder is from 0 to 150 mm; the diameter of the vortex finder varies from 10 to 50 mm and the range of the wall thickness of the vortex finder is from 1 to 30 mm. For each case, only one variable/dimension is changed while the rests are the same as the base hydrocyclone (Table 1).

Table 1
Geometry of the base hydrocyclone considered

Parameter	Symbol	Dimension
Diameter of the body	D_c	75 mm
Diameter of inlet	D_i	25 mm (same area quadrate is used)
Diameter of vortex finder	D_o	25 mm
Diameter of apex	D_u	12.5 mm
Length of vortex finder	L_v	50 mm
Thickness of vortex finder	D_v	1 mm
Length of cylindrical part	L_c	75 mm
Length of conical part	L_p	186 mm (included angle 20°)

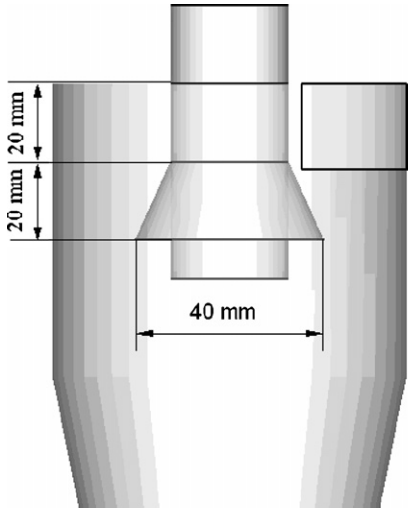

Fig. 2. Schematic representation of the new hydrocyclone considered (top part only).

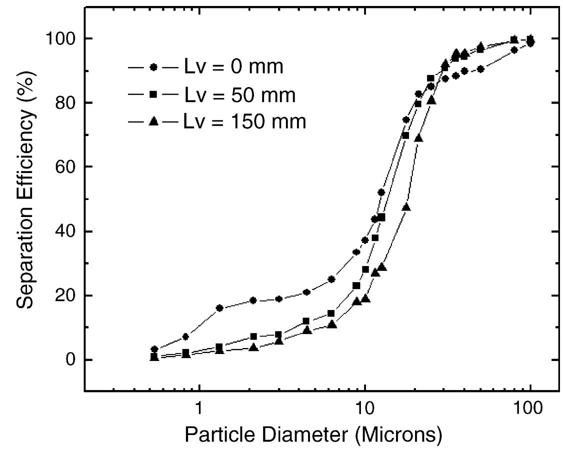

Fig. 3. The effect of vortex finder length on separation efficiency.

In addition, computation is also applied to a new design of vortex finder. Fig. 2 shows the geometry of this new design for vortex finder in detail. Its geometry and operated parameters keep the same as the base hydrocyclone except for the change of the vortex finder.

2.3. Model validation

It is necessary to validate the mathematical model before its application for numerical experiments. This is done by comparing the predicted and measured experimental flow fields including the tangential and axial velocity distributions at different axial locations; the measurements are taken from the work of Hsieh [7] involving the use of Laser Doppler Velocimetry (LDV). Moreover, the comparison between the experimental and numerical results has also been made for the performance in the hydrocyclone, such as pressure drop, volume split ratio and separation efficiency. The experimental results agree reasonably well with the calculated ones. The errors are less than 10% [16].

3. Results and discussion

3.1. Effect of vortex finder length

Figs. 3 and 4 show that the cut size d_{50} increases as vortex finder length increases and achieves the lowest with no vortex finder. Under these conditions there is, however, poor separation of coarse particles. The main reason is that large quantities of coarse particles bypass the separation process via the short circuit flow under the top cover and report to the overflow. Extension of the vortex finder, however, shortens the natural vortex in the cyclone body and reduces the opportunity of the fine particles to separate from the vortex. Therefore, a long vortex finder increases the cut size. This has been confirmed for plant hydrocyclones [1]. The empirical Plitt's model [18,19] predicts a similar trend, as shown in Fig. 4. However, quantitatively the present CFD model and the empirical model are different. The pressure drop increases with increasing the vortex finder length. The Plitt's model predicted a higher increase rate than that of the CFD model. The split ratio reported to the underflow reduces as the vortex finder length goes up, which is predicted by the Plitt's model, as shown in Fig. 4. However, the results of the CFD model shows the split ratio goes down firstly and then up with increasing the vortex finder length.

Plitt's model is formulated based on a large number of measurements and widely accepted as a design tool [3,20]. The agreement between the present CFD and empirical results confirms the general applicability of the proposed CFD model. However, the CFD model is superior to the empirical model because it is developed based on mechanistic understanding and can provide not only the macroscopic results of practical interest but also the internal flow as discussed below. It can lead to improved understanding and design of hydrocyclones.

Figs. 5 and 6 show the description of the fluid and solid flow in the hydrocyclones with $L_v = 0$ mm and $L_v = 150$ mm. It also

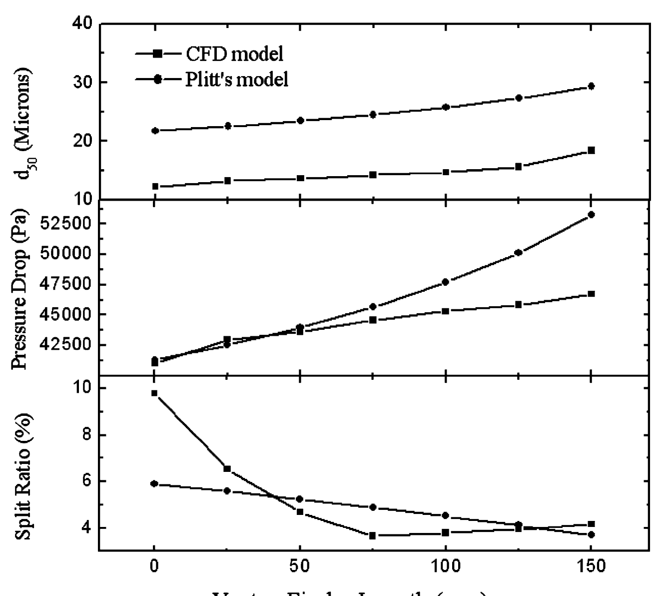

Fig. 4. The performance vs. vortex finder length: top, cut size; middle, pressure drop; bottom, split ratio.

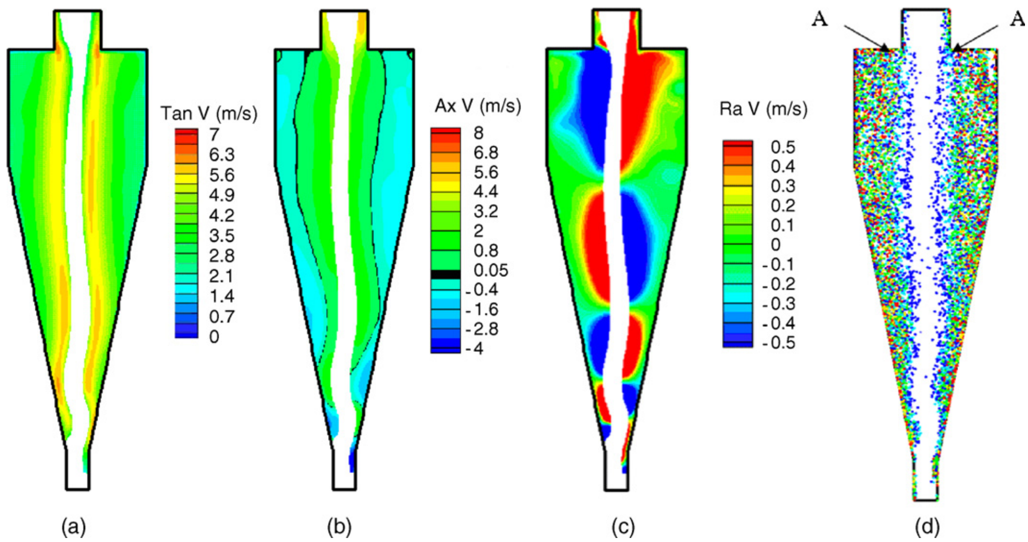

Fig. 5. Fluid and solid flow patterns in the hydrocyclone with $L_v = 0$ mm: (a) tangential velocity distribution where anti-clockwise is positive and clockwise is negative; (b) axial velocity distribution where upward is positive and downward is negative and the axial velocity in the black line equals zero; (c) radial velocity distribution where outward is positive and inward is negative; (d) particle size distribution where red, orange, green, cyan and blue respectively represent particles with five diameters, which are 2, 11.5, 21, 30.5 and 40 μm.

presents a snapshot of the transient simulation when the flow field is not sensitive to time, except for a small fluctuation caused by air core, which occupies the white region in the plot. Comparison of Figs. 5(a) and 6(a) indicates that the cyclone with no vortex finder has a lower tangential velocity in body, which weakens the vortex intensity and leads to a lower pressure drop. Figs. 5(b) and 6(b) plot the axial velocity distribution on the central vertical plane. The black line in the figures is the dividing line between the upward flow and the downward flow, which is called the locus of zero vertical velocity. The inner upward flow is more steady and strong in the cyclone with longer vortex finder than that in the cyclone with no vortex finder. It indicates the cyclone with no or shorter vortex finder has a better efficiency to collect particles. A large short circuit flow is found in the cyclone with $L_v = 150$ mm from the radial velocity distribution at Point A (Fig. 6(c)). Since the vortex finder is extended to the conical part and the separating space becomes small. Fluid and particles probably would rebound on the contracting wall to the inner upward flow, resulting in a strong short circuit flow. Moreover, it can be seen from Figs. 5 and 6 that the air core in the cyclone with no vortex finder is thinner than that in the cyclone with longer vortex finder, which results in a higher split ratio. Once the vortex finder exceeds the junction between the cylindrical and conical parts, however, the diameter of air core becomes small again since the separating space between the wall and the air core reduces, which results in an increased split ratio.

Figs. 5(d) and 6(d) show the spatial distribution of particles of different sizes in the hydrocyclones when $L_v = 0$ mm and

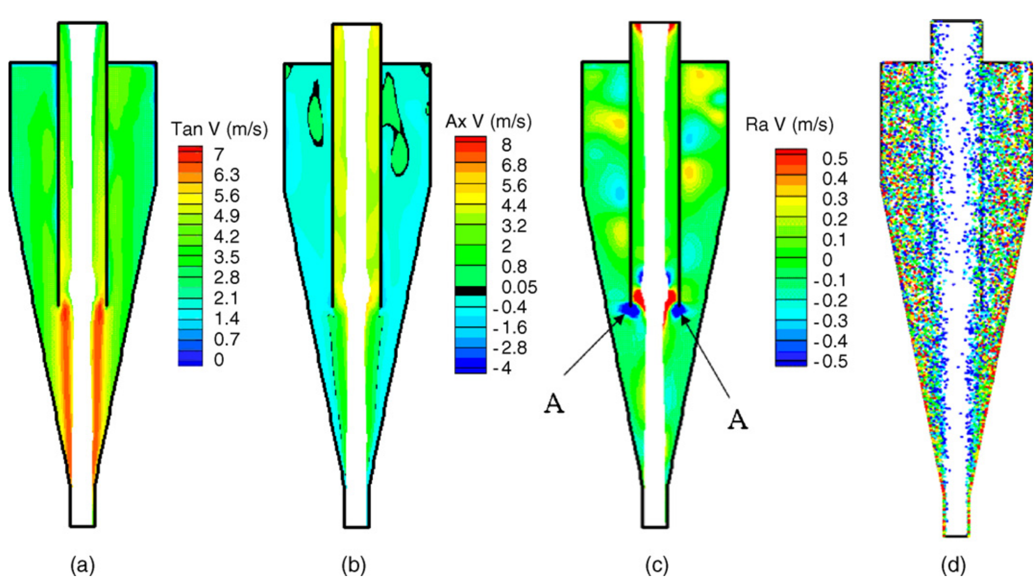

Fig. 6. Fluid and solid flow patterns in the hydrocyclone with $L_v = 150$ mm with figure description same as Fig. 5.

$L_v = 150$ mm respectively. Consistent with the physical experiment [7], particles of five different diameters were chosen to track their trajectories and distributions in a hydrocyclone, where red, orange, green, cyan and blue represent 2, 11.5, 21, 30.5 and 40 μm particles, respectively. In the hydrocyclone with $L_v = 0$ mm, most of particles go down with the downward flow and the fine particles (blue) concentrate around the air core (not shown in the figure to avoid mess), and escape from the vortex finder along the upward flow, whilst parts of coarse particles (yellow and red) escape to the overflow via the short circuit flow presented at Point A in the figure. That is why the hydrocyclone with no vortex finder has a good separation efficiency for fine particles but poor efficiency for coarse particles. In the hydrocyclone with $L_v = 150$ mm, coarse particles accumulate on the wall and collected from the underflow. On the other hand, fine particles enter the upward flow via the strong short circuit flow and escape from the vortex finder. This explains why a cyclone with a longer vortex finder is good at collecting coarse particles but has a poor collection efficiency for fine particles. Therefore, vortex finder in hydrocyclone cannot be too short or too long. Its bottom would be designed between the bottom of inlet and the junction of the cylindrical and conical part.

3.2. Effect of vortex finder diameter

The vortex finder diameter, i.e. the overflow diameter D_o, is another important dimension affecting the performance of hydrocyclones. Fig. 7 shows that the separation efficiency increases as the vortex finder diameter deceases. When the vortex finder diameter is lower than a critical value, which equals 30 mm for the hydrocyclone considered, however, the efficiency becomes zero. At the same time, both the pressure drop and the split ratio go down quickly (Fig. 8), which is similar to the results of Plitt's model.

Fig. 9(a) shows an extremely high tangential velocity in the cyclone with $D_o = 10$ mm compared with Fig. 10(a), which results in an extremely high-pressure drop. Moreover, air core

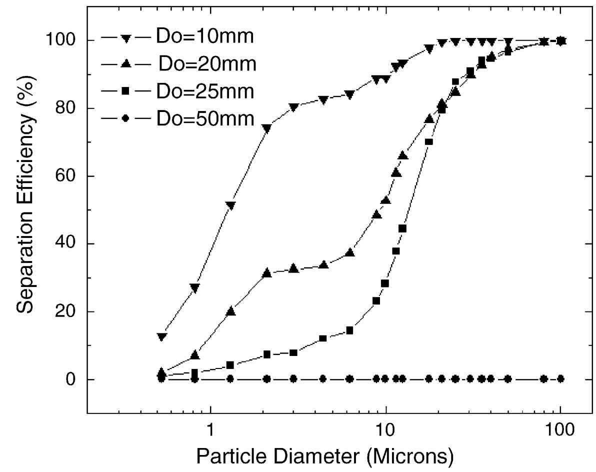

Fig. 7. The effect of vortex finder diameter on separation efficiency.

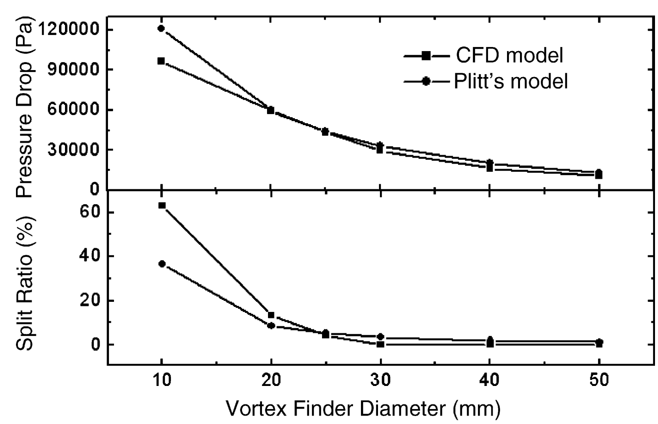

Fig. 8. The performance vs. vortex finder length: top, pressure drop; bottom, split ratio.

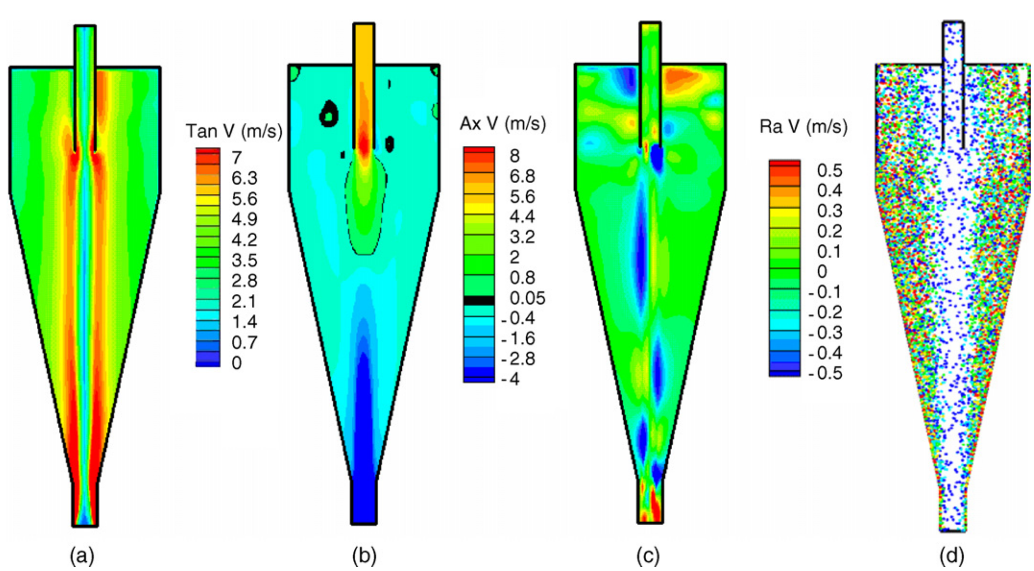

Fig. 9. Description of the fluid and solid flow in the hydrocyclone with $D_o = 10$ mm with figure description same as Fig. 5.

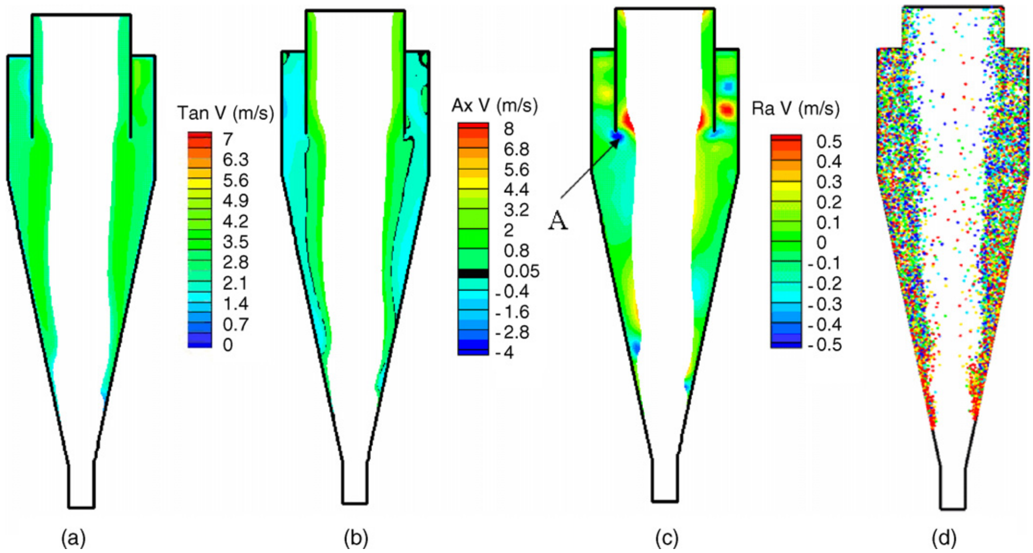

Fig. 10. Description of the fluid and solid flow in the hydrocyclone with $D_o = 50$ mm with figure description same as Fig. 5.

disappears in this cyclone. On the other hand, the diameter of air core in the cyclone with $D_o = 50$ mm exceeds the diameter of apex shown in Fig. 10; consequently the under flow equals zero. Thus it can be seen that the diameter of vortex finder is critical to the size of air core in hydrocyclones. In the axial velocity distribution on the vertical plane (Fig. 9(b)), there is a quite strong downward flow in the conical part of the cyclone with $D_o = 10$ mm, which indicates a good separation efficiency and a high split ratio. However, the "good" efficiency is not helpful, if the split ratio is too high. Figs. 9(c) and 10(c) show the radial velocity distribution, where Point A illustrates a strong short circuit flow in the cyclone with $D_o = 50$ mm. Since the space between the wall and the outside of vortex finder becomes smaller in the cyclone with a wider vortex finder, the collision between water streams after running about one circle with that just entering becomes more serious resulting in a stronger short circuit flow. That can explain why the separation efficiency in the cyclone with a thinner vortex finder is slightly higher than that with a wider one. Once the diameter of air core exceeds the diameter of apex, the efficiency and the split ratio become zero.

Figs. 9(d) and 10(d) show the distributions of different sized particles in the hydrocyclones with $D_o = 10$ mm and $D_o = 50$ mm. In the cyclone with thin vortex finder, only some fine particles (blue) move up with the upward flow in the inner region, while large particles (red and yellow) accumulate on the wall and are collected from the under flow. Since the air core is larger than the diameter of apex, in the cyclone with a wider vortex finder, all particles escape from the over flow. And some coarse particles concentrate on the wall of the conical part.

3.3. Effect of vortex finder thickness

The wall thickness of the vortex finder D_V may affect the flow and hence the performance of a cyclone [1]. However, to the authors' knowledge, its effect has not been quantified in the past. Therefore, as part of the present study, an attempt has been made to achieve this goal.

Fig. 11 shows that as the vortex finder thickness increases, the separation efficiency decreases obviously for coarse particles but remains at a same level for fine particles. Fig. 12 demonstrates that the pressure drop decreases to a minimum and then increases, and the split ratio gives an opposite trend with the increase of the wall thickness. The results indeed confirm that thickness of vortex finder affects the performance of hydrocyclone. However, it should be noted that the effect is relatively small, about 10% in pressure drop or in split ratio.

The observed effect can again be explained from the internal flow in the cyclone. Comparing Figs. 13(a), 14(a) and 15(a) suggests that the peak of the tangential velocity in the cyclone with $D_V = 1$ mm, i.e. the base case, is slightly higher than that in the other two. Correspondingly, the increase of the wall thickness can reduce the pressure drop. On the other hand, it decreases the annulus width for flow. When the thickness exceeds a critical value, the interaction among the fluids between the vortex finder and the body wall becomes serious, thereby raising the pressure drop again. Moreover, the eddy flow (to be discussed

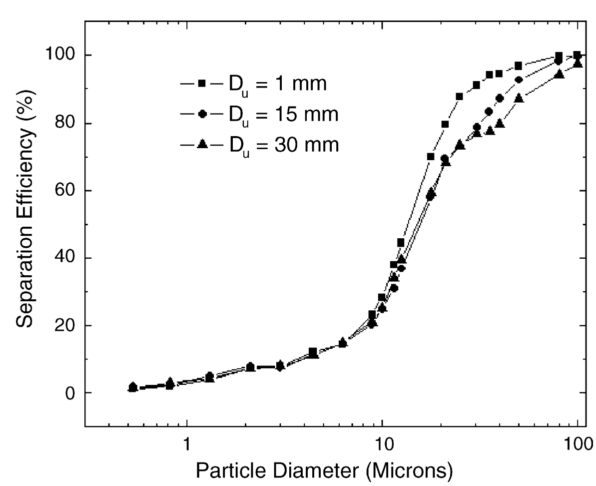

Fig. 11. The effect of vortex finder thickness on separation efficiency.

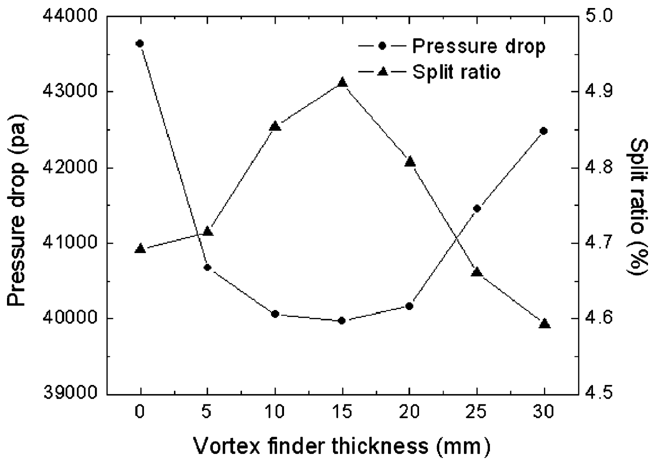

Fig. 12. The performance vs. vortex finder thickness.

in more detail in Section 3.4) in the cyclone with $D_v = 1$ mm (the base cyclone shown in Fig. 15(b)) is weakened or disappears for the cyclones with $D_v = 15$ and 30 mm shown in Figs. 13(b) and 14(b). This is good to the separation efficiency and pressure drop. However, it can be seen from the radial velocity distribution at Point A (Figs. 13(c), 14(c) and 15(c)) that the effect of the short circuit flow becomes more and more obvious as the wall thickness increases, which results in a lower separation efficiency for coarse particles.

3.4. New design for vortex finder

The previous numerical study [16] pointed out that the main reason for the short circuit flow in a hydrocyclone is the collision between water streams after running about one circle with that just entering. The flow enters the inlet and accelerates up to 1.5–2.0 times of the inlet velocity. Then the velocity decreases

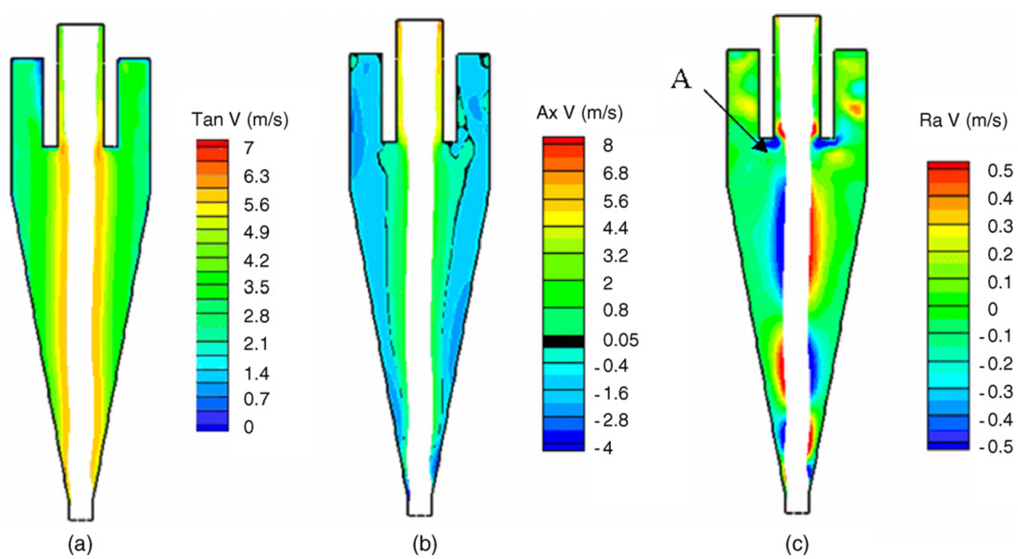

Fig. 13. Fluid flow patterns in the hydrocyclone with $D_v = 15$ mm, with figure description same as Fig. 5.

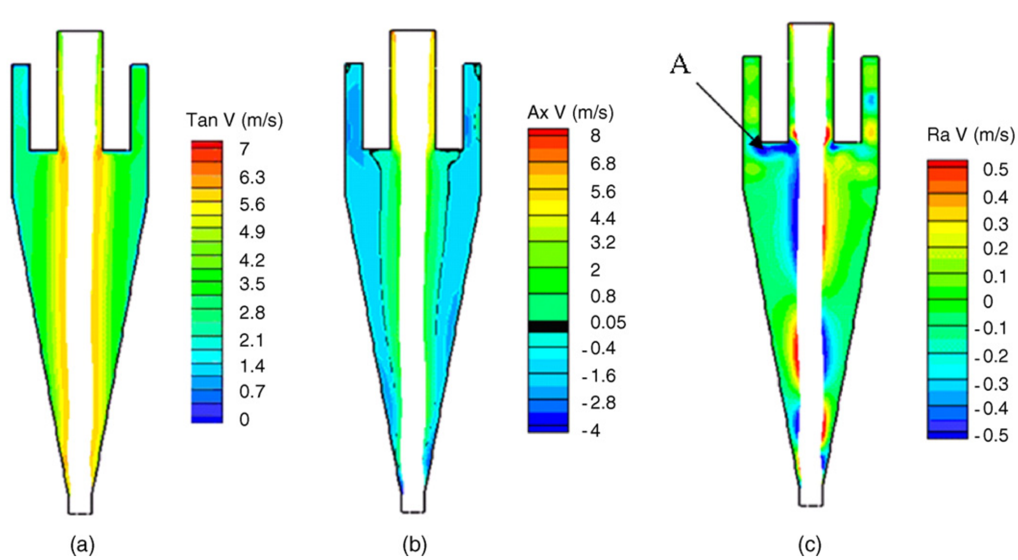

Fig. 14. Fluid flow pattern in the hydrocyclone with $D_v = 30$ mm, with figure description same as Fig. 5.

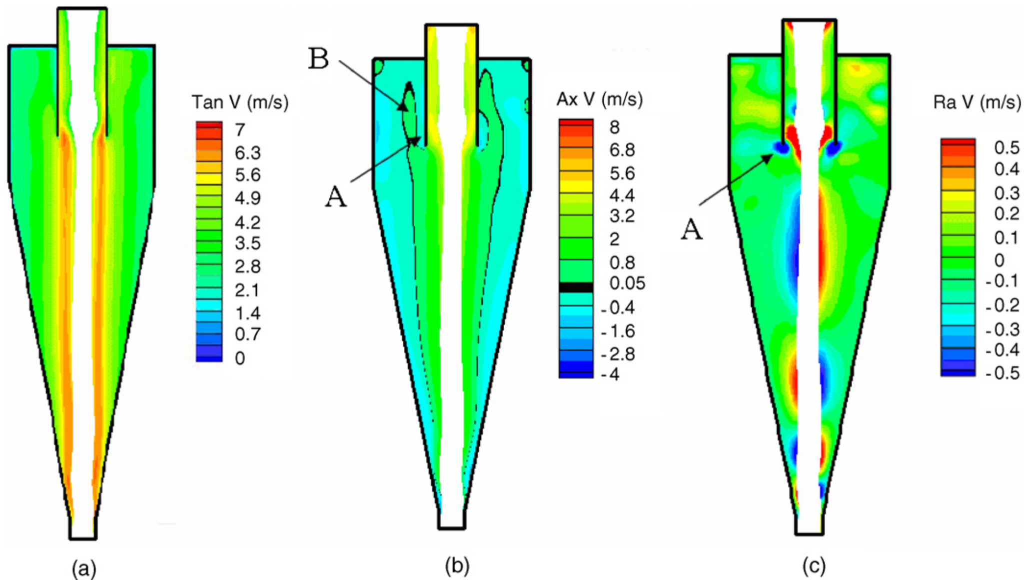

Fig. 15. Description of the fluid flow in the base hydrocyclone with figure description same as Fig. 5.

as the flow spins down along the wall. Before it goes below the vortex finder, the fluid flow collides with the follow-up flow and forms a chaotic flow close to the vortex finder outside wall and the flow velocity decreases sharply. As a result of the collision, a proportion of the feed liquid passes directly across the cyclone roof and down the outside wall of the vortex finder to join the overflow stream within the vortex finder, which is the short circuit flow (Point A), as shown in Fig. 15(b and c). At Point B, a vertical flow can exist in the region outside the outer wall of the vortex finder, which is called the eddy flow. It exists in the form of a recirculating eddy or eddies. The radial velocity distribution in the base hydrocyclone (Fig. 15(c)) indicates a zone under the vortex finder (Point A in the figure), where fluid directly flows into the vortex finder rather than moving down to the conical part and then flowing upward. The secondary flow including the short circuit and the eddy flow reduces the hydrocyclone performance [1]. The quantity of short circuit flow has been measured to be as much as 15% of the feed flow [21]. This behaviour would increase the loss of energy and decrease the efficiency apparently.

In order to overcome this problem, a new "mantle" type vortex finder is developed for hydrocyclone. A mantle is added to the base hydrocyclone, and other geometry and operated parameters can be same. For the present study, the detailed geometry is showed in Fig. 2. It is hoped that the mantle can lead the short circuit flow back to the main flow and the particles in the short circuit flow can continue to separate in the lower part of the hydrocyclone.

Fig. 16(a) shows the calculated tangential velocity distribution in this new hydrocyclone. Because of the presence of the mantle, the space for fluid flow is reduced. Therefore the collision between the water streams after running about one circle

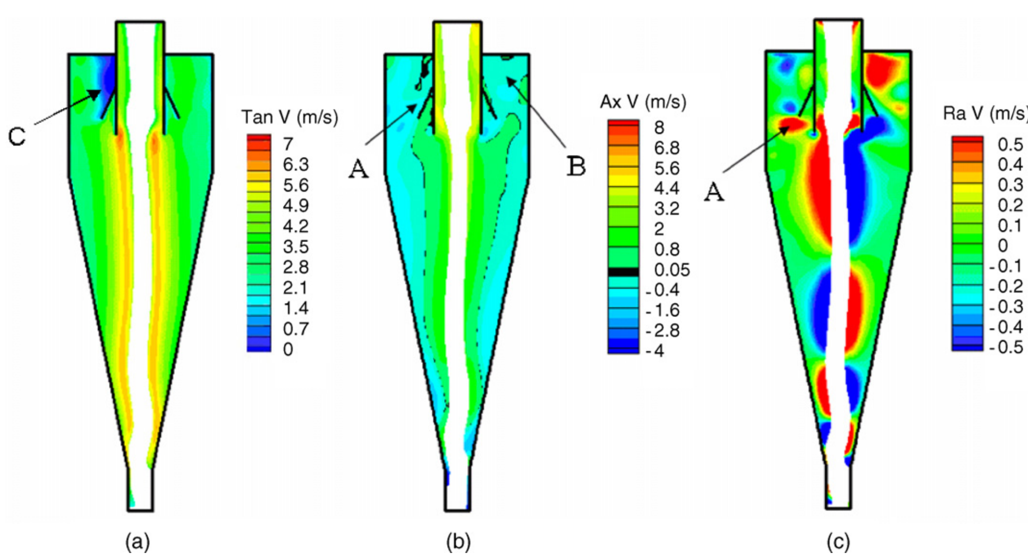

Fig. 16. Description of the fluid flow in the new hydrocyclone with figure description same as Fig. 5.

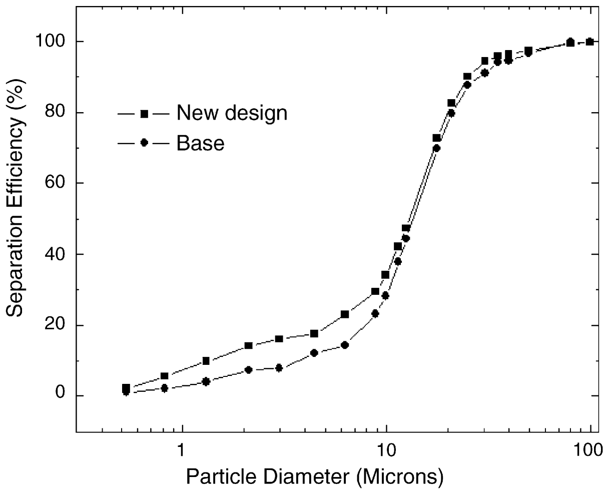

Fig. 17. Comparison of the separation efficiency between the base and new hydrocyclones.

with that just entering becomes more vigorous compared with the base cyclone. It forms a "stagnant zone" at Point C. In the "stagnant zone", the tangential velocity decreases to zero or even has a change in direction. The flow from inlet would pass around the "stagnant zone" and enter the main downward vertical flow. The zone enhances the collision among the fluid flows and dissipates energy. However, this zone also reduces the peak tangential velocity and weakens the intensity of the vortex. The energy drop of the vortex plays a crucial role in the total pressure drop in hydrocyclones. Therefore, the pressure drop in the cyclone with the mantle decreases from 43,638 to 39,628 Pa.

Fig. 16(b) plots the axial velocity distribution in the new hydrocyclone. It can be seen that the downward short circuit flow along the outer wall of vortex finder is led to the main flow (Point A). Particles in the short circuit flow can be separated in the lower part of the cyclone. At the same time, the eddy flow is weakened because of the effect of the mantle. The eddy flow presented at Point B almost disappears. The axial velocity distribution in the lower part of the cyclone is similar to the base cyclone. The radial velocity distribution (Fig. 16(c)) confirms the short circuit flow is weakened at Point A. Moreover, the inner upward flow is more instable in the new cyclone than that in the base cyclone.

Fig. 17 shows the comparison of the separation efficiency between the base and new hydrocyclones. The new cyclone has a better performance than the base one, particularly for fine particles. There are various methods that can increase the separation efficiency of a hydrocyclone, for example, increasing the length of cyclone, increasing inlet feed rate and so on. However, such methods usually reduce other performance. For example, increasing the length of body would result in a high consumption of raw materials and occupy more space. Decreasing the diameter of body would reduce the product capacity. Increasing inlet feed rate would cause a high-pressure drop. Therefore, a hydrocyclone has to be designed as a balance of the effects of all factors. The advantage of this new design is that it can increase separation efficiency and at the same time decrease the pressure drop. It should be useful in practice.

4. Conclusions

A numerical study has been made to examine the flow and performance of hydrocyclones with different vortex finder by means of the mathematical model proposed by Wang et al. [16,17]. The findings are summarized as follows:

(1) The separation efficiency decreases for fine particles and increases for relatively coarse particle as vortex finder length decreases because of the effect of the short circuit flow.
(2) A thin vortex finder can improve separation efficiency but results in an increased pressure drop. Once the diameter of vortex finder exceeds a critical value, the diameter of air core would be larger than that of the apex, which yields zero separation efficiency.
(3) As the wall thickness of the vortex finder increases, the effect of eddy flow is weakened but the effect of the short circuit flow is strengthened, which results in poor separation efficiency for coarse particles.
(4) A hydrocyclone with a mantle shape vortex finder can improve the hydrocyclone performance considerably. With this new design, the pressure drop can reduce by about 10%, and the collection efficiency increases for fine particles.

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